Psyc 3101, Homework 4

Your 3-year-old niece has a vocabulary of 500 words, which gives her a z-score of +1 for kids her age. Your neighbor's kid has a vocabulary of 200 words and a z-score of -2.

1. What's the standard deviation?

The z-scores differ by 3, meaning the kids differ by 3 standard deviations.
$3\sigma = 500-200 \rightarrow \sigma = \underline{100}$

2. What's the mean?

Using the first kid, $z = (X-\mu)/\sigma \rightarrow 1 = (500-\mu)/100 \rightarrow 100 = 500 - \mu \rightarrow \mu = \underline{400}$
Of course you get the same answer from the 2nd kid: $-2 = (200-\mu)/100 \rightarrow -200 = 200 - \mu \rightarrow \mu = 400$

A z-table gives the probability of a z-score greater than each listed value, in a normal distribution. People used these before computers, and there's one in the back of your textbook. Here's a z-table:

z	p(Z≥z)	z	p(Z≥z)	z	p(Z≥z)	z	p(Z≥z)
0	?	.5	.309	1.0	.159	1.5	.067
.1	.460	.6	.274	1.1	.136	1.6	.055
.2	.421	.7	.242	1.2	.115	1.7	.045
.3	.382	.8	.212	1.3	.097	1.8	.036
.4	.345	.9	.184	1.4	.081	1.9	.029

3. What value belongs in the question mark?

.5, because z = 0 at the mean, and 50% of a normal distribution is above the mean

4. What's the probability of a z-score greater than -1?

Because the normal distribution is symmetric, this is the same as the probability of a z-score less at 1.
$P(z<1) = 1 - P(z\geq1) = 1 - .159 = \underline{.841}$

5. What's the probability of a z-score between .5 and 1?

30.9% of scores are above .5. Of those, 15.9% are above 1, and the rest are between .5 and 1.
30.9% – 15.9% = 15% or .15

6. When you get off an international flight to Singapore, they take your temperature and if it's over 39° C they quarantine you to keep diseases out of the country. Of course even healthy people have natural variability in temperature, following a normal distribution with mean 37° and standard deviation 2.5°. What percentage of healthy people are mistakenly quarantined? (This question is only partially made up.)

The z-score for 39° equals (39° – 37°)/2.5° = .8. Therefore anyone with a z-score above .8 for healthy people will be quarantined. The probability of a z-score above .8 is .212 or 21.2%.

A lottery ticket can pay $5 or $100, or it can pay nothing. The probability of winning $5 is 1 in 10. The probability of winning $100 is 1 in 100.

7. What is the probability of a ticket paying nothing?
Probabilities have to sum to 1. 1 – 1/10 – 1/100 = .89 or 89%

8. What is the expected value of how much a ticket will pay?

$\sum x \cdot p(x) = \$0 \cdot 89\% + \$5 \cdot 10\% + \$100 \cdot 1\% = \$0 + 50¢ + \$1 = \$1.50$

9. If the state sells a million tickets at $1 each, about how much profit will it make?
It pays out an average of $1.50 per ticket, or $1,500,000 total ($1.50 · 1,000,000 = $1,500,000). It takes in $1,000,000 from selling the tickets. $1,000,000 – $1,500,000 = –$500,000 (the state loses money).

10. Calculate the variance of {11, 18, 21, 14, 9, 15}, treated as a population.

$$\mu = \frac{\sum X}{N} = \frac{11+18+21+14+9+15}{6} = 14.67$$

$$X - \mu = \{-3.67, 3.33, 6.33, -.67, -5.67, .33\}$$

$$(X-\mu)^2 = \{13.44, 11.11, 40.11, .44, 32.11, .11\}$$

$$\sum (X-\mu)^2 = 97.33$$

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N} = \frac{97.33}{6} = 16.22$$

11. Calculate the variance of the same numbers, treated as a sample.

The mean and the sum of squares are the same as above. The only difference is in dividing by *n*-1 instead of by *N*.

$$s^2 = \frac{\sum (X-M)^2}{n-1} = \frac{97.33}{5} = 19.47$$

12. A population has a variance of 8. Imagine you collect a large number of independent samples, each with 30 subjects, and compute the sample variance of each sample. What would you expect the average of your sample variances to be?

Since the sample variance is an unbiased estimator of the population variance, the average of the sample variances should be about 8. (It won't be exactly 8 if you get a finite number of samples, but as the number of samples approaches infinity the average sample variance will approach exactly 8.)