Lecture 11: Single-Sample t-test
Notation and Equations

Notation

μ_0: The value of the population mean assumed by the null hypothesis in a basic t-test

t: t statistic – an inferential statistic measuring the reliability of the difference between *M* and μ_0

df: degrees of freedom; determines which t distribution to use

Formula for t statistic (single-sample t-test). The t statistic tells how reliable the difference is between a sample mean (*M*) and a hypothesized population mean (μ_0). It's our first example of a purely inferential statistic, because its value doesn't have any physical meaning. If $t = 3$, this doesn't mean three **of** anything; it just means we can be fairly confident that μ_0 is not the true population mean.

The simplest way to think of *t* is as the number of standard errors between *M* and μ_0. To get this, we take the difference, $M - \mu_0$, and divide it by the standard error, $s/\sqrt{n}$.

$$t = \frac{M - \mu_0}{s/\sqrt{n}} \tag{1}$$

Recall that the standard error is the typical distance between *M* and μ. Therefore, *t* tells us whether μ_0 is a reasonable value for μ by telling us whether $M - \mu_0$ is a reasonable value for $M - \mu$. For example, if $t = 3$, this means that *M* is three times as far from μ_0 as you'd expect.

Degrees of freedom for a basic t-test. The probability distribution (or likelihood function) of a t statistic under the null hypothesis depends on its degrees of freedom. Therefore, you need to know how many degrees of freedom you have before you know which distribution to use.

Degrees of freedom originates as a property of chi-square distributions. Recall that the sampling distribution of the sample variance, s^2, is a chi-square distribution with $n - 1$ degrees of freedom. This value comes from the denominator, $n - 1$, in the formula for s^2, because that's the number of squares that are being averaged to get s^2. Since the definition of *t* includes *s*, *t* also has $n - 1$ degrees of freedom.

$$df = n - 1 \tag{3}$$