Lecture 12: Hypothesis Testing
Notation and Equations

Notation

H_0: Null hypothesis

H_1: Alternative hypothesis

α: alpha level; the Type I error rate the researcher decides is acceptable; also the cutoff for the p-value between retaining and rejecting the null hypothesis

t_{crit}, etc.: critical value. t_{crit} is the critical value when your test statistic is t. Soon we'll learn about a test statistic named F, and its critical value is F_{crit}, and so on.

Critical value for one-tailed tests. The critical value is determined by our choice of Type I error rate. The Type I error rate is the probability, when the null hypothesis is true, of incorrectly rejecting it. We reject the null hypothesis whenever the test statistic is beyond the critical value, so the Type I error rate is the probability of all values beyond the critical value. By probability, we mean probability according to the null hypothesis.

$$p_{H_0}(t > t_{\text{crit}}) = \alpha \tag{1}$$

Note that this formula is written in terms of t_{crit}, but it applies to other types of hypothesis tests as well. Soon we'll learn about F tests, and the same formula will apply for F and F_{crit}.

Critical values for two-tailed tests. If we want to be able to detect effects in either direction, e.g. a mean either less than or greater than μ_0, then we need to set up the hypothesis test so that the null hypothesis is rejected whenever the test statistic is too small or too big. This is done by using two critical values, one positive and one negative, with the rule:

$$\begin{gathered}\text{Reject } H_0 \text{ if } t < -t_{\text{crit}} \text{ or } t > t_{\text{crit}} \\ \text{i.e.} \\ \text{Reject } H_0 \text{ if } |t| > t_{\text{crit}}\end{gathered} \tag{2}$$

As above, the Type I error rate is the probability, according to the null hypothesis, of getting a value beyond the critical values. In this case, that probability is $p_{H_0}\left(|t| > t_{\text{crit}}\right)$, or $p_{H_0}(t < -t_{\text{crit}}) + p_{H_0}(t > t_{\text{crit}})$. The probabilities in the two tails are the same, because t distributions are always symmetric. Therefore, each probability needs to equal $\alpha/2$ in order for the total probability (i.e., the Type I error rate) to equal α. We can write this in a few different ways:

$$
\begin{aligned}
&p_{H_0}\left(|t| > t_{\text{crit}}\right) = \alpha \\
&p_{H_0}\left(t < -t_{\text{crit}}\right) = \tfrac{\alpha}{2} \\
&p_{H_0}\left(t > t_{\text{crit}}\right) = \tfrac{\alpha}{2}
\end{aligned}
\qquad (3)
$$