

Lecture 15: Three Views of Hypothesis Testing

What are we doing?

In science, we run lots of experiments

In some cases, there's an "effect"; in others there's not

Some manipulations have an impact; some don't

Some drugs work; some don't

Goal: Tell these situations apart, using the sample data

If there is an effect, reject the null hypothesis

If there's no effect, retain the null hypothesis

Challenge

Sample is imperfect reflection of population, so we will make mistakes

How to minimize those mistakes?

Analogy: Prosecution

Think of cases where there is an effect as "guilty"

No effect: "innocent" experiments

Scientists are prosecutors

We want to prove guilt

Hope we can reject null hypothesis

Can't be overzealous

Minimize convicting innocent people

Can only reject null if we have enough evidence

How much evidence to require to convict someone?

Tradeoff

Low standard: Too many innocent people get convicted (Type I Error)

High standard: Too many guilty people get off (Type II Error)

Binomial Test

Example: Halloween candy

Sack holds boxes of raisins and boxes of jellybeans

Each kid blindly grabs 10 boxes

Some kids cheat by peeking

Want to make cheaters forfeit candy

How many jellybeans before we call kid a cheater?

Work out probabilities for honest kids

Binomial distribution

Don't know distribution of cheaters

Make a rule so only 5% of honest kids lose their candy

For each *individual* above cutoff

Don't know for sure they cheated

Only know that if they were honest, chance would have been less than 5% that they'd have so many jellybeans

Therefore, our rule catches as many cheaters as possible while limiting Type I errors (false convictions) to 5%

Testing the Mean

Example: IQs at various universities

Some schools have average IQs (mean = 100)

Some schools are above average

Want to decide based on n students at each school

Need a cutoff for the sample mean

Find distribution of sample means for Average schools

Set cutoff so that only 5% of Average schools will be mistaken as Smart

Unknown Variance: t-test

Want to know whether population mean equals μ_0

$H_0: \mu = \mu_0$

$H_1: \mu \neq \mu_0$

Don't know population variance

Don't know distribution of M under H_0

Don't know Type I error rate for any cutoff

Use sample variance to estimate standard error of M

Divide $M - \mu_0$ by standard error to get t

We know distribution of t exactly

Two-tailed Tests

Often we want to catch effects on either side

Split Type I Errors into two critical regions

Each must have probability $\alpha/2$

An Alternative View: p-values

p-value

Probability of a value equal to **or more extreme** than what you actually got

Measure of how consistent data are with H_0

$p > \alpha$

t is within t_{crit}

Retain null hypothesis

$p < \alpha$

t is beyond t_{crit}

Reject null hypothesis; accept alternative hypothesis

Three Views of Inferential Statistics

Effect size & confidence interval

Values of μ_0 that don't lead to rejecting H_0

Test statistic & critical value

Measure of consistency with H_0

p-value & α

Type I error rate

Can answer hypothesis test at any level

Result predicted by H_0 vs. confidence interval

Test statistic vs. critical value

p vs. α