Psych 2111
Prof. Matt Jones

Lecture 20: Factorial ANOVA

Multiple Independent Variables
- Simple (one-way) ANOVA tells whether groups differ
  - Compares levels of a single independent variable
- Sometimes we have multiple IVs
  - <u>Factors</u>
  - Subjects divided in multiple ways
    - Training type & testing type
  - Not always true independent variables
    - Undergrad major & sex
  - Some or all can be within-subjects (gets more complicated)
    - Memory drug & stimulus type
- Dependent variable measured for all combinations of values
- Factorial ANOVA
  - How does each factor affect the outcome?
  - Extends ANOVA in same way regression extends correlation

Basic Approach
- Calculate sum of squares for each factor
  - Variability explained by that factor
  - Essentially by averaging all data for each level of that factor
- Separate hypothesis test for each factor
  - Convert SS to mean square
  - Divide by MS_{residual} to get F

<u>Interactions</u>
- Effect of one factor may depend on level of another
  - Pick any two levels of Factor A, find difference of means, compare across levels of Factor B
- Testable in same way as <u>main effect</u> of each factor
  - $SS_{\text{interaction}}$, $MS_{\text{interaction}}$, F, p
- Can have higher-order interactions
  - Interaction between Factors A and B depends on C
- Partitioning variability
  - $SS_{\text{total}} = SS_{\text{A}} + SS_{\text{B}} + SS_{\text{C}} + SS_{\text{A:B}} + SS_{\text{A:C}} + SS_{\text{B:C}} + SS_{\text{A:B:C}} + SS_{\text{residual}}$

Example: Memory and Brain Injury

	Brain Injury			
Delay	None	Occipital	MTL	Mean
Short	78%	65%	73%	72%
Long	66%	53%	37%	52%
Difference	12%	12%	36%	
Mean	72%	59%	55%	62%

Rule for an interaction:

Pick any two levels of Factor A (A_1, A_2) and any two levels of Factor B (B_1, B_2)

There's an interaction if $M_{A_1,B_1} - M_{A_1,B_2} \neq M_{A_2,B_1} - M_{A_2,B_2}$

Equivalently: $M_{A_1,B_1} - M_{A_2,B_1} \neq M_{A_1,B_2} - M_{A_2,B_2}$

Testing main effects and interactions:

Effect	*SS*	*df*	*MS*	*F*	*p*
Delay	6000	1	6000	12.91	.0007
Injury	3160	2	1580	3.40	.041
Delay:Injury	1920	2	960	2.07	.136
Residual	25094	54	464.7		

Logic of Sum of Squares

Total sum of squares: $\sum\left(X - \overline{M}\right)^2$

Null hypothesis assumes all data are from same population

Expected value of $\left(X - \overline{M}\right)^2$ is σ^2 for each raw score

No matter how we break up SS_{total}, every individual square has expected value σ^2

$SS_{\text{treatment}}$, $SS_{\text{interaction}}$, SS_{residual} are all sums of numbers with expected value σ^2

Every *MS* has expected value σ^2

Average of many numbers that all have expected value σ^2

$E(MS_{\text{treatment}})$, $E(MS_{\text{interaction}})$, $E(MS_{\text{residual}})$ all equal σ^2, according to H_0

If H_0 false, then $MS_{\text{treatment}}$ and $MS_{\text{interaction}}$ tend to be larger

F is sensitive to such an increase